Webinar: Höhere Mathematik 1
Thema: Differentialrechnung

Notiztitel 06.09.2016

1) $f(x) = x^5 - 6x^4 + 5x^2 + 3$

$$\boxed{f'(x) = n \cdot x^{n-1}}$$

$f'(x) = 5x^4 - 6 \cdot 4x^3 + 5 \cdot 2x$

$f'(x) = 5x^4 - 24x^3 + 10x$

Tangente:

$g(x) = f(x_0) + f'(x_0)(x - x_0)$ $\quad x_0 = 1$

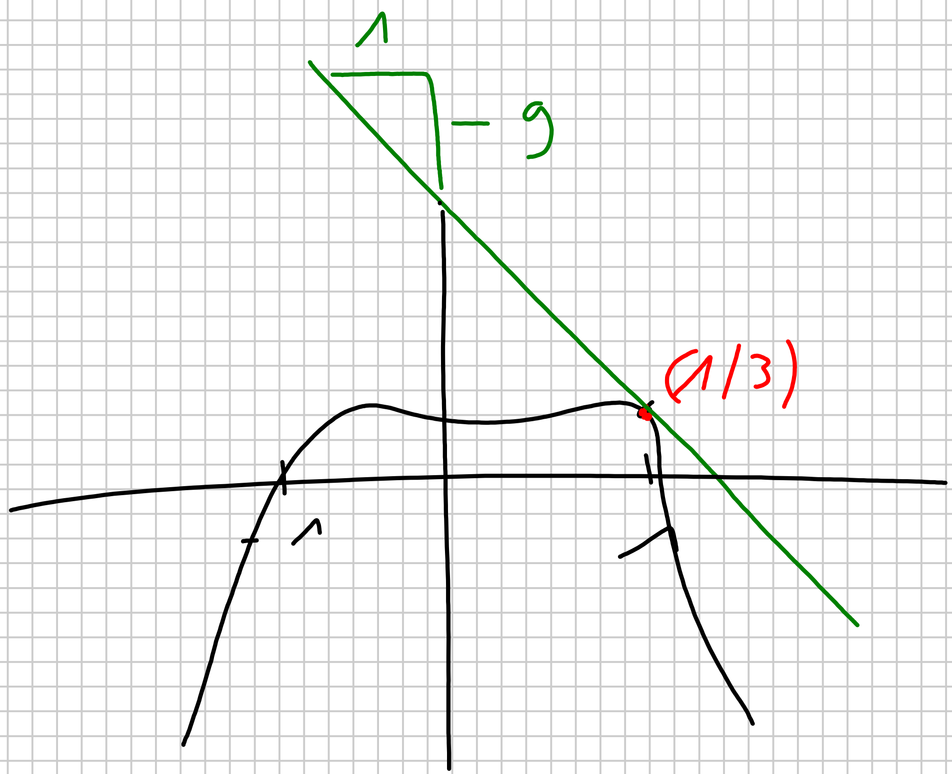
$g(x) = f(1) + f'(1)(x - 1)$

$f(1) = 3$ $\quad x_0 = 1 \quad P(1/3)$

$f'(1) = -9$

$g(x) = 3 - 9 \cdot (x-1)$

$\boxed{g(x) = -9x + 12}$



$f(x) = \sqrt{2x^2 + 9}$

$f(x) = (2x^2 + 9)^{1/2}$

Äußere Ableitung

$\frac{1}{2} \cdot (2x^2 + 9)^{-1/2}$

Innere Ableitung:

$2x^2 + 9' = 4x$

$f'(x) = \frac{1}{2} (2x^2 + 9)^{-1/2} \cdot 4x$

$f'(x) = \frac{1}{2 \cdot \sqrt{2x^2 + 9}} \cdot 4x = \frac{2x}{\sqrt{2x^2 + 9}}$

$g(x) = f(x_0) + f'(x_0) \cdot (x - x_0)$

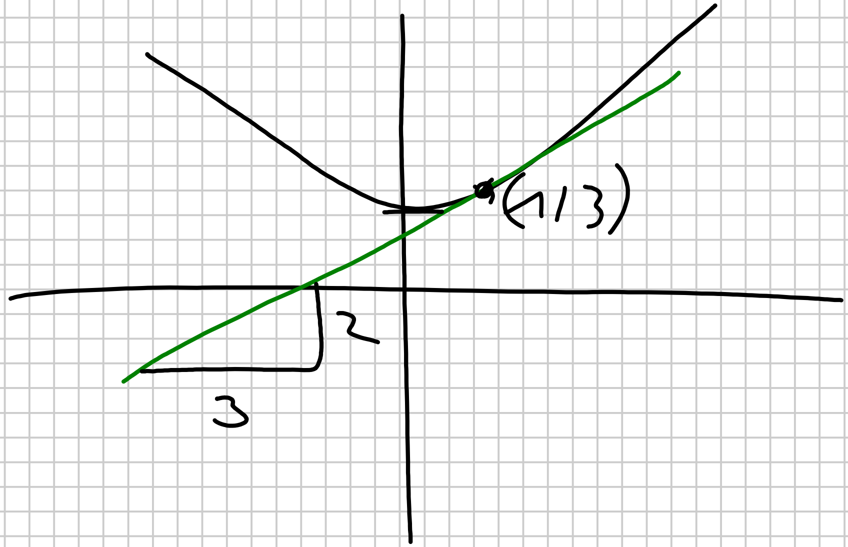
$g(x) = f(1) + f'(1) \cdot (x - 1)$

$f(1) = 3$ $\qquad P(1|3)$

$f'(1) = \frac{2}{3}$

$g(x) = 3 + \frac{2}{3} \cdot (x - 1)$

$$\boxed{g(x) = \frac{2}{3}x + \frac{7}{3}}$$



$$f(x) = (4x + 3 \cdot \cos^2 x)^5$$

<u>Äußere Ableitung</u>

$$5 \cdot (4x + 3 \cdot \cos^2 x)^4$$

<u>Innere Ableitung</u>

$$4x + 3 \cdot \cos^2 x'$$

$$4x' = 4$$

$$3 \cdot \cos^2 x = 3 \cdot (\cos x)^2$$

$$6 \cdot (\cos x)$$

$\cos x' = -\sin x$

$\left[4 + 6 \cdot \cos x \cdot (-\sin x)\right] \cdot 5 \cdot (4x + 3 \cdot \cos^2 x)^4$

$$\boxed{f'(x) = 5 \cdot (4x + 3 \cdot \cos^2 x)^4 \cdot (4 - 6 \cdot \cos x \cdot \sin x)}$$

2) $f(x) = x^3 \cdot (x-3)$

Monotonie

1. Ableitung

$f(x) = x^4 - 3x^3$

$f'(x) = 4x^3 - 9 \cdot x^2 = 0$

$x^2(4x - 9) = 0$

$x^2 = 0 \qquad x_1 = +\sqrt{0} \qquad x_2 = -\sqrt{0} \qquad x_{1,2} = 0$

$4x - 9 = 0$

$\boxed{x_3 = \frac{9}{4}}$

<u>Bereiche</u>:

$x < 0$

$0 < x < \frac{9}{4}$

$x > \frac{9}{4}$

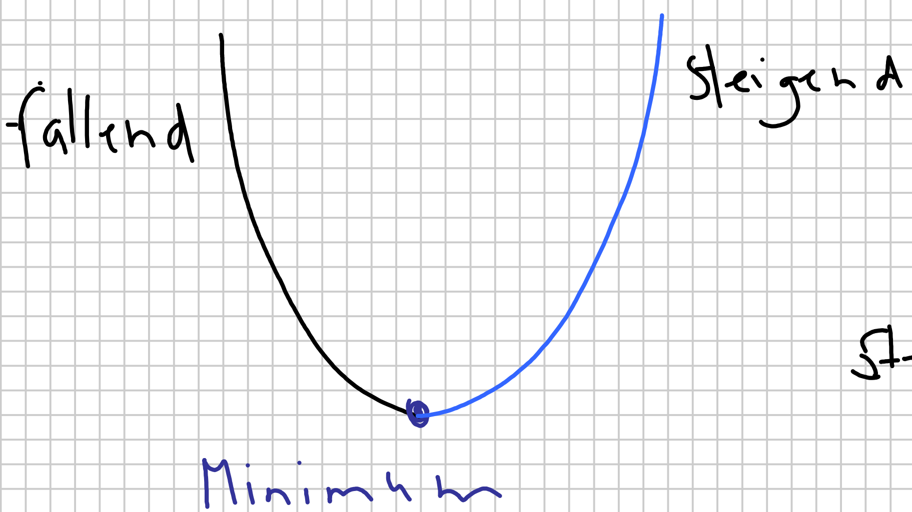
$f'(-1) = -13$ $\quad$ $f'(x) < 0$ monoton fallend

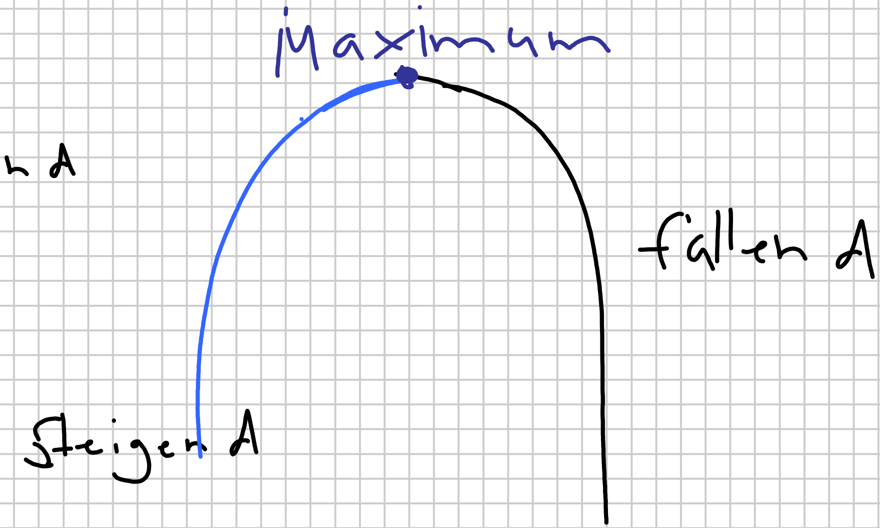
$f'(2) = -4$ $\quad$ $f'(x) < 0$ monoton fallend

$f'(3) = 27$ $\quad$ $f'(x) > 0$ monton steigend

$(-\infty; \frac{9}{4})$

$$f'(x) = \begin{cases} < 0 & \text{für } x < 0 \quad \text{monoton fallend} \\ < 0 & \text{für } 0 < x < \frac{9}{4} \quad \text{monoton fallend} \\ > 0 & \text{für } x > \frac{9}{4} \quad \text{monoton steigend} \end{cases}$$





Extremwerte:

$f'(x) = x^2(4x - 9) \overset{!}{=} 0$ nullstellen

$\boxed{x_{1,2} = 0 \quad x_3 = 9/4}$

$f''(x) = 12x^2 - 18x$

$f''(0) = 0$ -> keine

$f''\left(\frac{9}{4}\right) = 20{,}25$ $f''(x) > 0$ -> <u>Minimum</u>

$f''(x) > 0$ -> Minimum

$f''(x) < 0$ -> Maximum

Wendepunkte

$f''(x) = 12x^2 - 18x = 0$ Nullstellen bestimmen

$x(12x - 18) = 0$

$\boxed{x_1 = 0}$

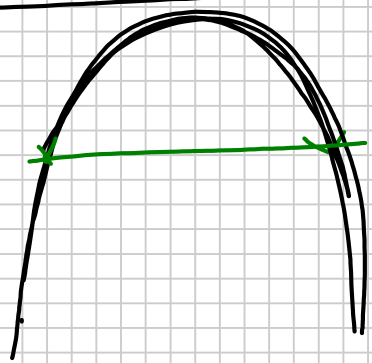
$12x - 18 = 0 \quad \rightarrow \quad \boxed{x_2 = 1{,}5}$

$f'''(x) = 24x - 18$

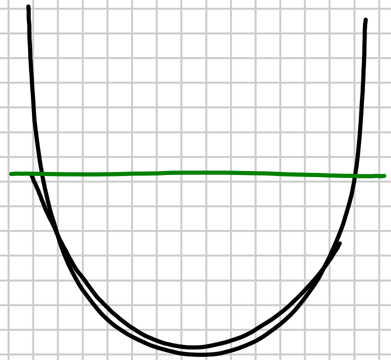
$f'''(0) = -18$ $\quad f'''(x) < 0 \rightarrow$ links nach rechts $(0/0)$

$f'''(1,5) = 18$ $\quad f'''(x) > 0 \rightarrow$ rechts nach links $(1,5/-5,01)$

Konkav/konvex



Konkav



konvex

$f''(x) = 12x^2 - 18x = 0$ Nullstellen bestimmen

$x(12x - 18) = 0$

$x_1 = 0 \qquad x_2 = 1{,}5$

$x < 0$

$0 < x < 1{,}5$

$x > 1{,}5$

$f''(x) = 12x^2 - 18x$

$f''(x_0) > 0$ konvex

$f''(x_0) < 0$ konkav

$f''(-1) = 30$ streng konvex

$f''(1) = -6$ streng konkav

$f''(2) = 12$ streng konvex

$$f''(x) = \begin{cases} >0 & x<0 \quad \text{streng konvex} \\ <0 & 0<x<1{,}5 \quad \text{streng konkav} \\ >0 & x>1{,}5 \quad \text{streng konvex} \end{cases}$$