Webinar: Höhere Mathematik 1
Thema: Differentialrechnung

Notiztitel 06.09.2016

1) $f(x) = x^5 - 6x^4 + 5x^2 + 3$

$$\boxed{f'(x) = n \cdot x^{n-1}}$$

$$f'(x) = 5x^4 - 6 \cdot 4x^3 + 5 \cdot 2x$$

$$f'(x) = 5x^4 - 24x^3 + 10x$$

<u>Tangente</u>:

$$g(x) = f(x_0) + f'(x_0)(x - x_0) \qquad x_0 = 1$$

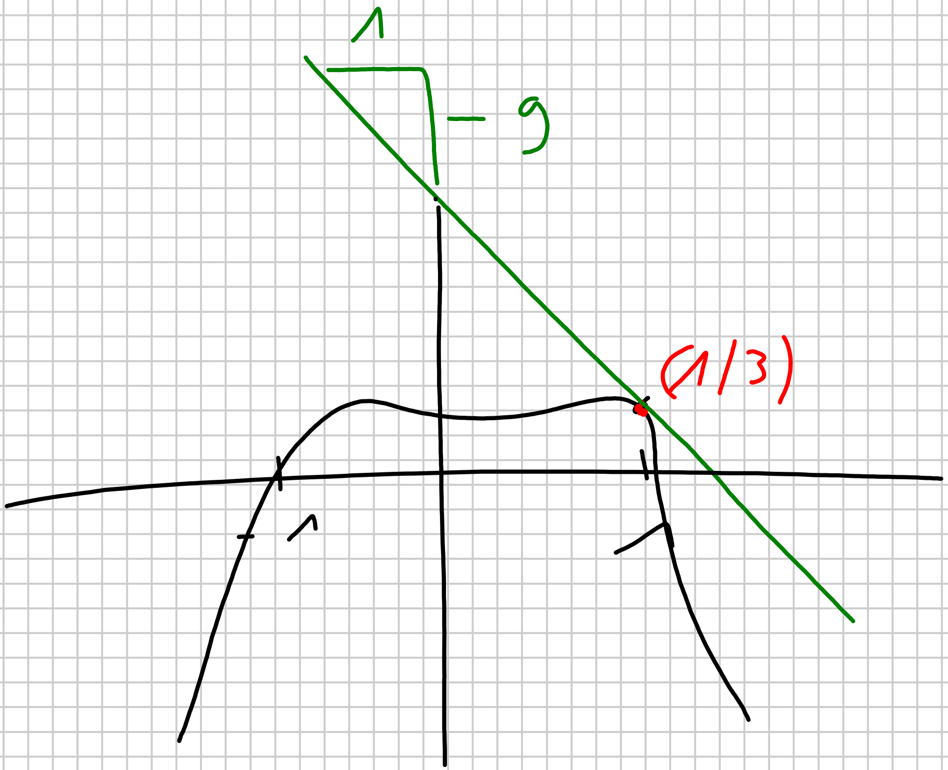
$$g(x) = f(1) + f'(1)(x - 1)$$

$f(1) = 3$ $\quad x_0 = 1 \quad P(1/3)$

$f'(1) = -9$

$g(x) = 3 - 9 \cdot (x - 1)$

$$\boxed{g(x) = -9x + 12}$$



$$f(x) = \sqrt{2x^2 + 9}$$

$$f(x) = (2x^2 + 9)^{1/2}$$

Äußere Ableitung

$$\frac{1}{2} \cdot (2x^2 + 9)^{-1/2}$$

Innere Ableitung:

$$2x^2 + 9' = 4x$$

$$f'(x) = \frac{1}{2}(2x^2 + 9)^{-1/2} \cdot 4x$$

$$f'(x) = \frac{1}{2 \cdot \sqrt{2x^2 + 9}} \cdot 4x = \frac{2x}{\sqrt{2x^2 + 9}}$$

$$g(x) = f(x_0) + f'(x_0) \cdot (x - x_0)$$

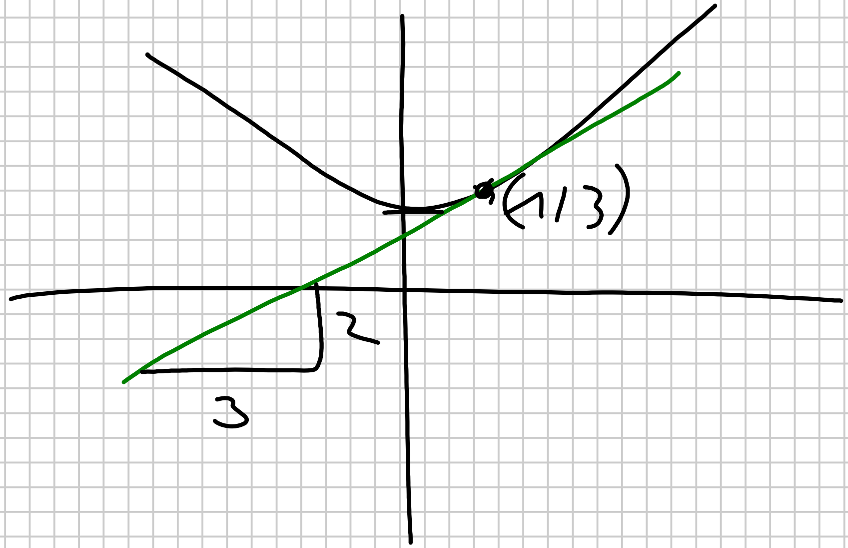
$$g(x) = f(1) + f'(1) \cdot (x - 1)$$

$$f(1) = 3 \qquad P(1|3)$$

$$f'(1) = \frac{2}{3}$$

$$g(x) = 3 + \frac{2}{3} \cdot (x - 1)$$

$$\boxed{g(x) = \frac{2}{3}x + \frac{7}{3}}$$



$$f(x) = (4x + 3\cdot\cos^2 x)^5$$

Äußere Ableitung

$$5\cdot(4x + 3\cdot\cos^2 x)^4$$

Innere Ableitung

$$4x + 3\cdot\cos^2 x\,'$$

$$4x' = 4$$

$$3\cdot\cos^2 x = 3\cdot(\cos x)^2$$

$$6\cdot(\cos x)$$

$$\cos x' = -\sin x$$

$$\left[4 + 6\cdot\cos x\cdot(-\sin x)\right]\cdot 5\cdot\left(4x+3\cdot\cos^2 x\right)^4$$

$$\boxed{f'(x) = 5\cdot\left(4x+3\cdot\cos^2 x\right)^4\cdot\left(4-6\cdot\cos x\cdot\sin x\right)}$$

2) $f(x) = x^3 \cdot (x-3)$

Monotonie

1. Ableitung

$f(x) = x^4 - 3x^3$

$f'(x) = 4x^3 - 9 \cdot x^2 = 0$

$x^2(4x - 9) = 0$

$x^2 = 0$ $\quad x_1 = +\sqrt{0} \quad x_2 = -\sqrt{0} \quad \boxed{x_{1,2} = 0}$

$4x - 9 = 0$

$$\boxed{x_3 = \frac{9}{4}}$$

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Bereiche :

$x < 0$

$0 < x < \frac{9}{4}$

$x > \frac{9}{4}$

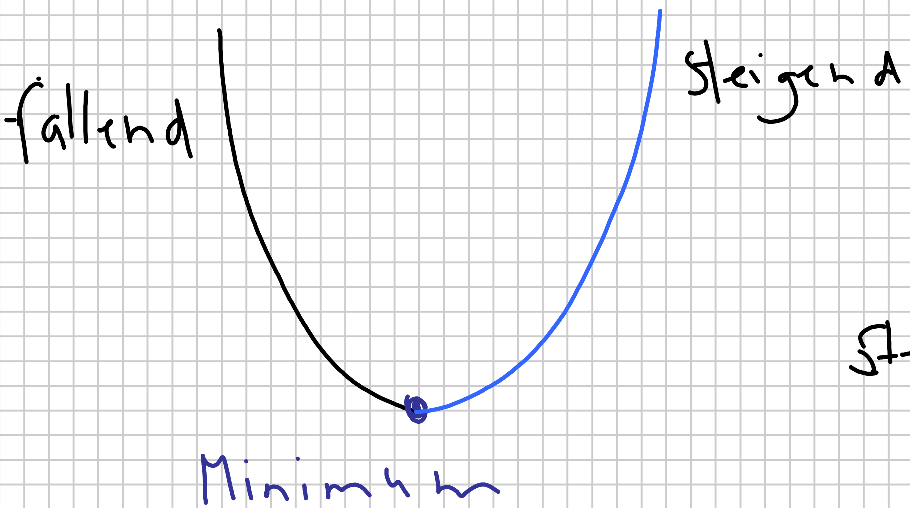
$f'(-1) = -13 \qquad f'(x) < 0$ monoton fallend

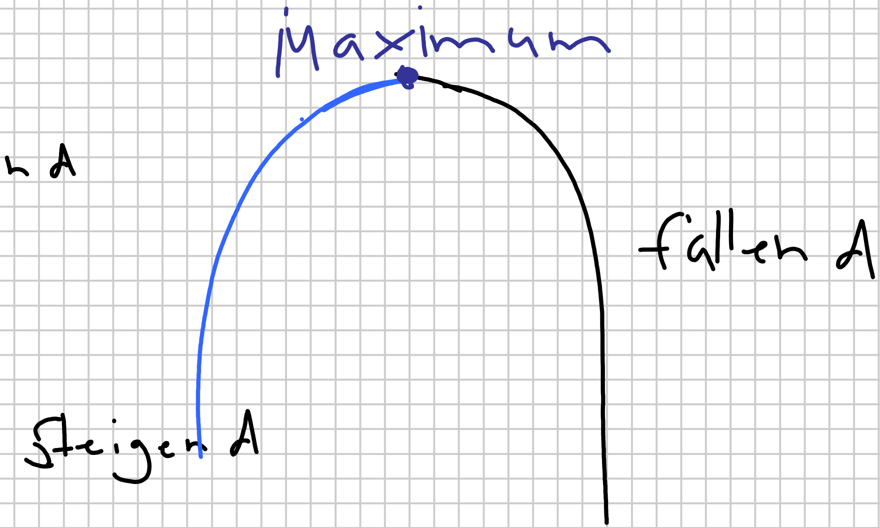
$f'(2) = -4 \qquad f'(x) < 0$ monoton fallend

$f'(3) = 27 \qquad f'(x) > 0$ monoton steigend

$\left(-\infty; \frac{9}{4}\right)$

$$f'(x) = \begin{cases} < 0 & \text{für } x < 0 \quad \text{monoton fallend} \\ < 0 & \text{für } 0 < x < \frac{9}{4} \quad \text{monoton fallend} \\ > 0 & \text{für } x > \frac{9}{4} \quad \text{monoton steigend} \end{cases}$$





Extremwerte:

$f'(x) = x^2(4x-9) = 0$ Nullstellen

$\boxed{x_{1,2} = 0 \quad x_3 = 9/4}$

$f''(x) = 12x^2 - 18x$

$f''(0) = 0$ -> keine

$f''\left(\frac{9}{4}\right) = 20{,}25$ $f''(x) > 0$ -> Minimum

$f''(x) > 0$ -> Minimum

$f''(x) < 0$ -> Maximum

Wendepunkte

$f''(x) = 12x^2 - 18x = 0$ Nullstellen bestimmen

$x(12x - 18) = 0$

$\boxed{x_1 = 0}$

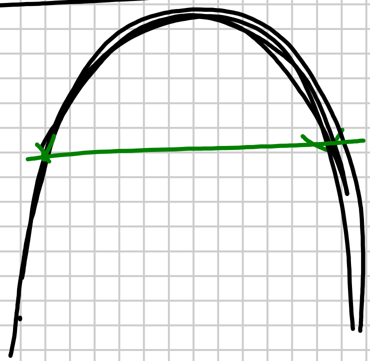
$12x - 18 = 0 \rightarrow \boxed{x_2 = 1{,}5}$

$f'''(x) = 24x - 18$

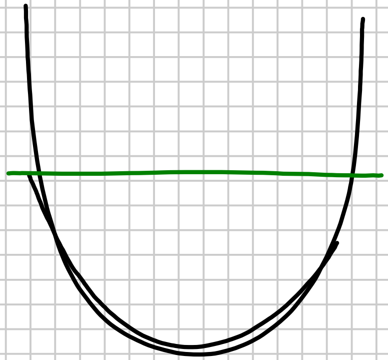
$f'''(0) = -18$ $\quad f'''(x) < 0 \to$ links nach rechts $(0/0)$

$f'''(1{,}5) = 18$ $\quad f'''(x) > 0 \to$ rechts nach links $(1{,}5/-5{,}06)$

Konkav/konvex



Konkav



konvex

$$f''(x) = 12x^2 - 18x = 0$$

Nullstellen bestimmen

$$x(12x - 18) = 0$$

$$x_1 = 0 \qquad x_2 = 1{,}5$$

$x < 0$

$0 < x < 1{,}5$

$x > 1{,}5$

$f''(x) = 12x^2 - 18x$

$f''(x_0) > 0$ konvex

$f''(x_0) < 0$ konkav

$f''(-1) = 30$ streng konvex

$f''(1) = -6$ streng konkav

$f''(2) = 12$ streng konvex

$$f''(x) = \begin{cases} > 0 & x < 0 \quad \text{streng konvex} \\ < 0 & 0 < x < 1{,}5 \quad \text{streng konkav} \\ > 0 & x > 1{,}5 \quad \text{streng konvex} \end{cases}$$